Consideremos la ecuación de onda electromagnética

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_x(z,t)$$

Hemos visto que una solucion es

$$E_x(z,t) = E_0 \operatorname{sen}(kz-\omega t)$$

En esta ocasión proponemos una solución

$$E_x(z,t) = E_0 \exp\left[-\frac{1}{2\sigma^2}(z-ct)^2\right]$$

Comprobación:

$$\frac{\partial}{\partial z} E_x(z,t) = E_0 \exp\left[-\frac{1}{2\sigma^2}(z-ct)^2\right]\left(\frac{-1}{2\sigma^2}\right) 2(z-ct)$$

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{\partial}{\partial z}\left\{ E_x(z,t)\left[-\frac{(z-ct)}{\sigma^2}\right]\right\}$$

$$= E_x(z,t)\left(\frac{-1}{\sigma^2}\right) + \left[\frac{-(z-ct)}{\sigma^2}\right]^2 E_x(z,t)$$

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \left[-\frac{1}{\sigma^2} + \frac{(z-ct)^2}{\sigma^2} \right] E_x(z,t)$$

Ahora

$$\frac{\partial}{\partial t} E_x(z,t) = E_0 \exp\left[-\frac{1}{2\sigma^2} (z-ct)^2 \right] \left(\frac{-1}{\sigma^2} \right) (z-ct)(-c)$$

$$= (-c) \left[-\frac{(z-ct)}{\sigma^2} \right] E_x(z,t)$$

$$\frac{\partial^2}{\partial t^2} E_x(z,t) = (-c) \; E_x(z,t) \left[+\frac{c}{\sigma^2} \right] + (-c)^2 \left[-\frac{(z-ct)}{\sigma^2} \right]^2 E_x(z,t)$$

$$= -\frac{c^2}{\sigma^2} E_x(z,t) + c^2 \left[\frac{(z-ct)^2}{\sigma^2} \right] E_x(z,t)$$

$$\frac{\partial^2}{\partial t^2} E_x(z,t) = c^2 \left\{ -\frac{1}{\sigma^2} + \frac{(z-ct)^2}{\sigma^2} \right\} E_x(z,t)$$

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = +\frac{1}{c^2} \frac{\partial^2}{\partial t^2} E_x(z,t)$$

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_x(z,t) \qquad E_x(z,t) = E_0 \exp\left[-\frac{1}{2\sigma^2}(z-ct)^2\right]$$

Ya que estamos convencidos de esta solucion, consideremos ahora una solucion con pdtd

$$\frac{\partial}{\partial z} E_x(z,t) = \frac{E_x(z+\Delta z/2, t) - E_x(z-\Delta z/2, t)}{\Delta z}$$

$$z = k\Delta z \qquad t = n\Delta t \qquad (z,t) = (k\Delta z, n\Delta t)$$

$$\frac{\partial}{\partial z} E_x(z,t) = \frac{E_x^n(k+1/2) - E_x^n(k-1/2)}{\Delta z}$$

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{E_x^n(k+1) - E_x^n(k)}{(\Delta z)^2} - \frac{E_x^n(k) - E_x^n(k-1)}{(\Delta z)^2}$$

$$= \frac{E_x^n(k+1) - 2E_x^n(k) - E_x^n(k-1)}{(\Delta z)^2}$$

$$\frac{\partial}{\partial t} E_x(z,t) = \frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t}$$

$$\frac{\partial^2}{\partial t^2} E_x(z,t) = \frac{E_x^{n+1}(k) - 2E_x^{n}(k) + E_x^{n-1}(k)}{(\Delta t)^2}$$

así tenemos

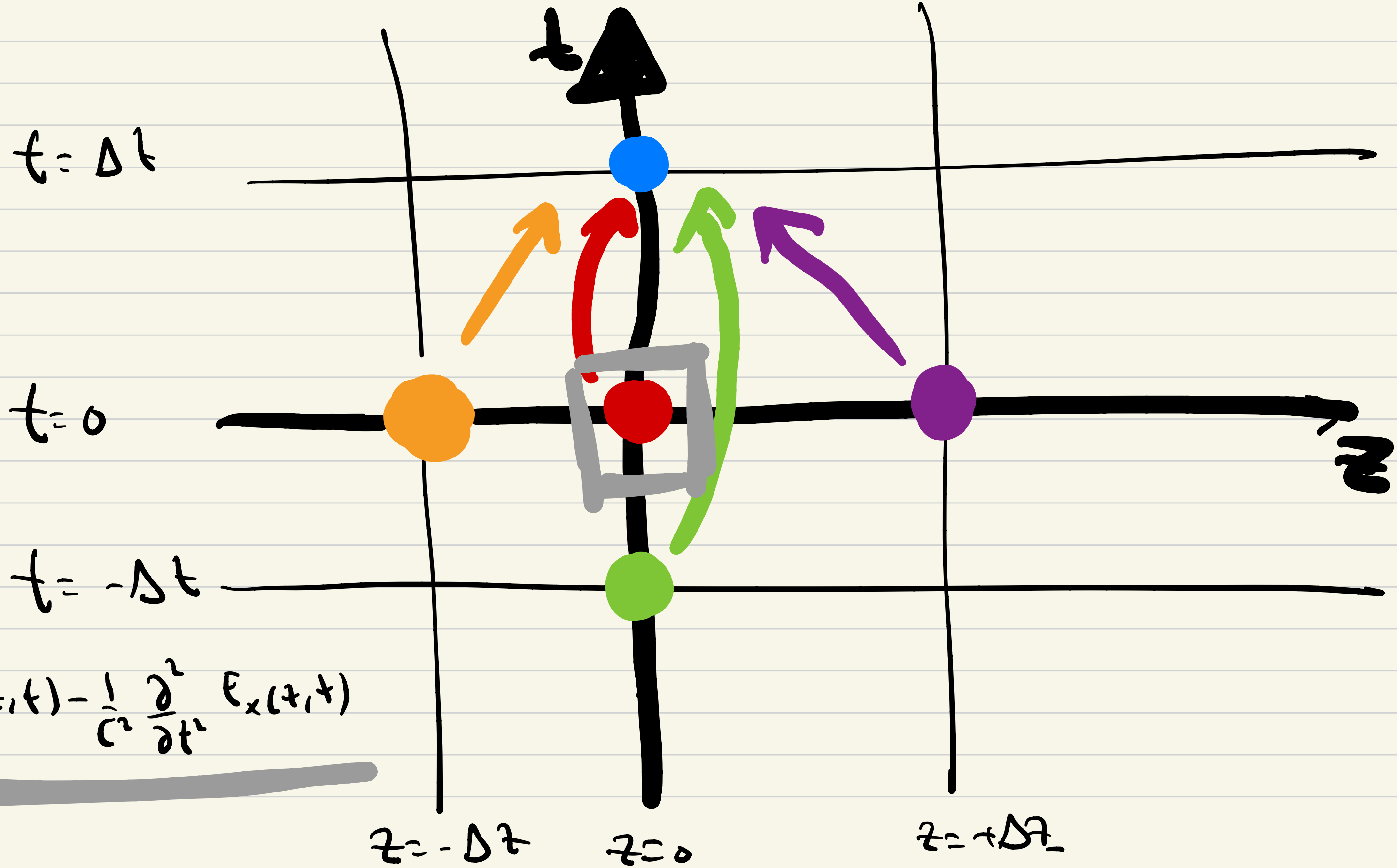
$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E_x(z,t)$$

$$\frac{E_x^{n}(k+1) - 2E_x^{n}(k) + E_x^{n}(k-1)}{(\Delta z)^2} = \frac{1}{c^2}\left[\frac{E_x^{n+1}(k) - 2E_x^{n}(k) + E_x^{n-1}(k)}{(\Delta t)^2}\right]$$

$$E_x^{n+1}(k) = +2E_x^{n}(k) - E_x^{n-1}(k) + \frac{c^2}{\Delta z^2}(\Delta t)^2\left[E_x^{n}(k+1) - 2E_x^{n}(k) + E_x^{n}(k-1)\right]$$

$$\Delta t = \frac{1}{2}\frac{\Delta z}{c}$$

$$E_x^{n+1}(k) = +2E_x^n(k) - E_x^{n-1}(k) + \frac{c^2}{\Delta z^2}(\Delta t)^2\left[E_x^n(k+1) - 2E_x^n(k) + E_x^n(k-1)\right]$$

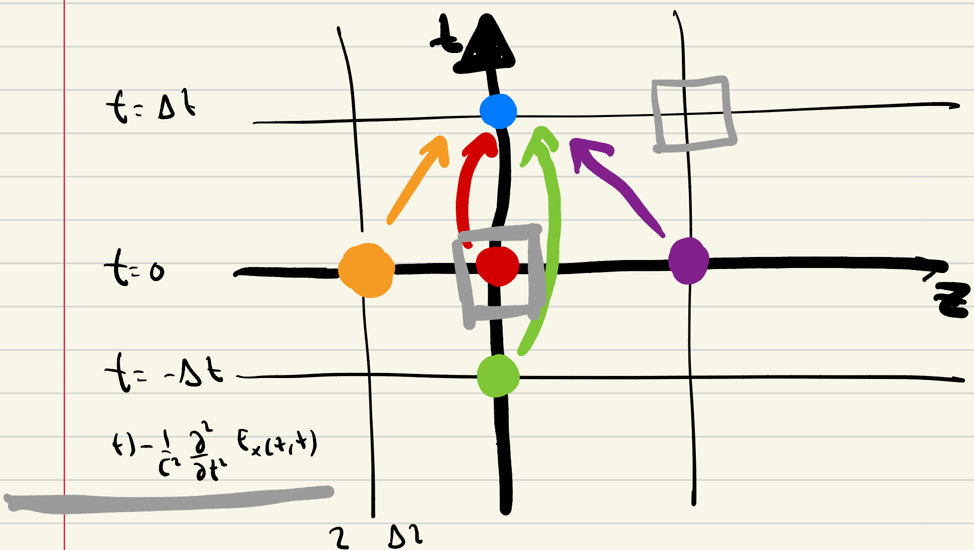


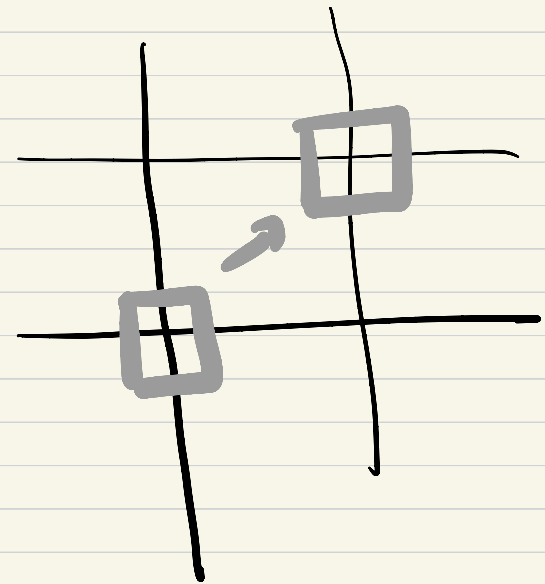
$$\frac{\partial^2}{\partial z^2}E_x(z,t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E_x(z,t)$$

Vamos a considerar la ecuación

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_x(z,t)$$

$$(z,t) = [(k+1)\Delta z, (n+1)\Delta t]$$





$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E_x(z,t) \qquad (z,t) = [(k+1)\Delta z, (n+1)\Delta t]$$

$$\frac{\partial}{\partial z} E_x(z,t) = \frac{E_x^{n+1}(k+3/2) - E_x^{n+1}(k-1/2)}{\Delta z}$$

$$\frac{\partial^2}{\partial z^2} E_x(z,t) = \frac{E^{n+1}(k+2) - E^{n+1}(k+1)}{(\Delta z)^2} - \frac{E_x^{n+1}(k+1) - E_x^{n+1}(k)}{(\Delta z)^2}$$

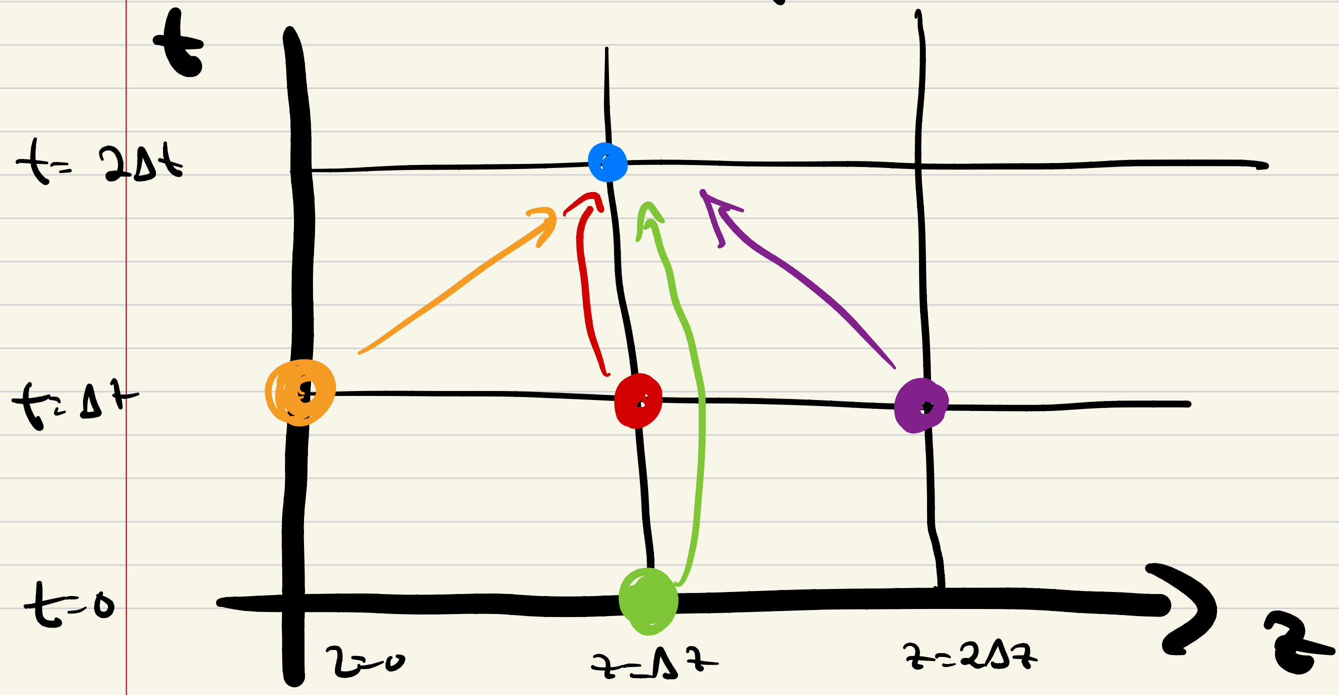
$$= \frac{E_x^{n+1}(k+2) - 2E^{n+1}(k+1) + E_x^{n+1}(k)}{(\Delta z)^2}$$

$$\frac{\partial}{\partial t} E_x(z,t) = \frac{E_x^{n+3/2}(k+1) - E_x^{n-1/2}(k+1)}{\Delta t}$$

$$\frac{\partial^2}{\partial t^2} E_x(z,t) = \frac{E_x^{n+2}(k+1) - 2E_x^{n+1}(k+1) + E_x^{n}(k+1)}{(\Delta t)^2}$$

$$\frac{E_x^{n+1}(k+2) - 2E^{n+1}(k+1) + E_x^{n+1}(k)}{(\Delta z)^2} = \frac{1}{c^2}\left[\frac{E_x^{n+2}(k+1) - 2E_x^{n+1}(k+1) + E_x^{n}(k+1)}{(\Delta t)^2}\right]$$

$$\underline{E_x^{n+2}(k+1)} = \underline{2E_x^{n+1}(k+1)} - \underline{E_x^{n}(k+1)} + \left(\frac{c\,\Delta t}{\Delta z}\right)^2\left[\underline{E_x^{n+1}(k+2)} - \underline{2E_x^{n+1}(k+1)} + \underline{E_x^{n+1}(k)}\right]$$



$$E_x(z,t) = E_0 \exp\left[-\frac{1}{2\sigma^2}(z-ct)^2\right]$$

Si $\underline{\underline{t=0}}$

$$E_x(z,0) = E_0 \exp\left[-\frac{1}{2\sigma^2} z^2\right]$$

Si $t = \Delta t$

$$E_x(z,\Delta t) = E_0 \exp\left[-\frac{1}{2\sigma^2}(z-c\Delta t)^2\right]^2$$