"El saber de mis hijos hará mi grandeza"

Complex band structure of thermal wave crystals: The plane-wave method

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Heat reduction by thermal wave crystals

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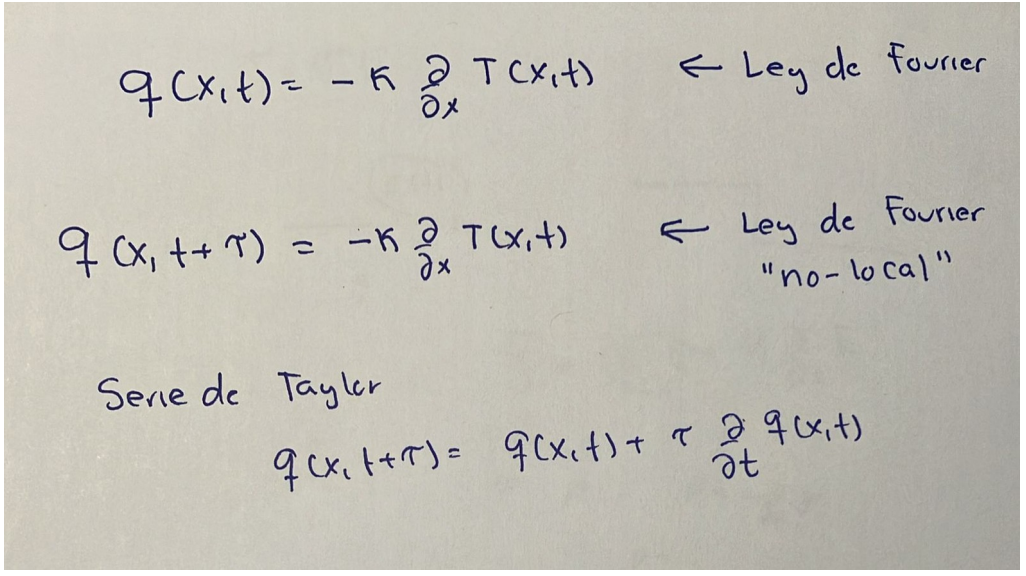

1. Introduction

Traditional Fourier conduction law with implicit assumption of instantaneous thermal propagation is no longer applicable under specific conditions such as ultralow temperature, micro scale and biological tissues. In 1958, Cattaneo [1] and Vernotte [2] separately proposed a model with a time lag between the heat flux vector and the temperature gradient. In the one-dimensional (1D) case, the Cattaneo-Vernotte (CV) heat-conduction model can be written as

$$q + \tau_q \frac{\partial q}{\partial t} = -\kappa \frac{\partial T}{\partial x}, \tag{1}$$

where q and T are heat flux and temperature, respectively; τ_q is the relaxation time for the phonon collision; and κ is the thermal conductivity. The equation of energy conservation is given by [3]

$$\frac{\partial q}{\partial x} = -\rho c_p \frac{\partial T}{\partial t} + Q, \tag{2}$$

where Q is the internal energy generation rate; ρ is the mass density; and c_p is the specific heat. Substitution of Eq. (1) into Eq. (2) yields [3]

$$\frac{1}{\tau_q}\frac{\partial T}{\partial t} + \frac{\partial^2 T}{\partial t^2} = \frac{\kappa}{\rho c_p \tau_q}\frac{\partial^2 T}{\partial x^2} + \frac{1}{\kappa}\left(Q + \tau_q \frac{\partial Q}{\partial t}\right). \tag{3}$$

PHYSICS

Observation of second sound in a rapidly varying temperature field in Ge

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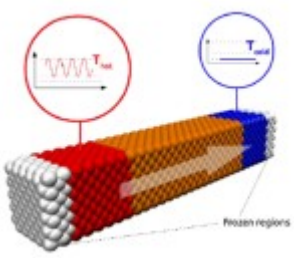
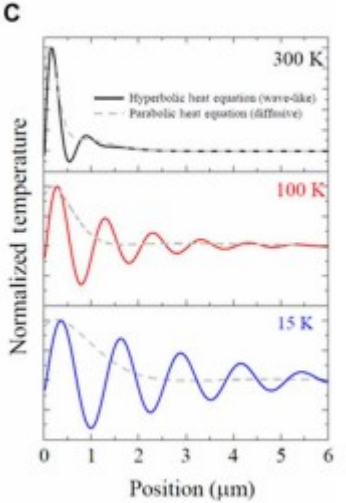
Second sound is known as the thermal transport regime where heat is carried by temperature waves. Its experimental observation was previously restricted to a small number of materials, usually in rather narrow temperature windows. We show that it is possible to overcome these limitations by driving the system with a rapidly varying temperature field. High-frequency second sound is demonstrated in bulk natural Ge between 7 K and room temperature by studying the phase lag of the thermal response under a harmonic high-frequency external thermal excitation and addressing the relaxation time and the propagation velocity of the heat waves. These results provide a route to investigate the potential of wave-like heat transport in almost any material, opening opportunities to control heat through its oscillatory nature.

$$\tau_{ss}\frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} - \alpha\nabla^2 T = \frac{1}{\rho C_p}\left(S(\boldsymbol{r},t) + \tau_{ss}\frac{\partial S(\boldsymbol{r},t)}{\partial t}\right)$$





$$\frac{1}{\tau_q}\frac{\partial T}{\partial t}+\frac{\partial^2 T}{\partial t^2}=\frac{\kappa}{\rho c_p \tau_q}\frac{\partial^2 T}{\partial x^2}+\frac{1}{\kappa}\left(Q+\tau_q\frac{\partial Q}{\partial t}\right).$$

2. Problem formulation

Consider a periodically layered structure with bilayer unit-cells as shown in Fig. 1. Each unit-cell consists of layer (sub-cell) A with thickness l_A and layer B with thickness l_B (the unit-cell's thickness $l=l_A+l_B$). All material properties $\{\kappa, \tau_q, \rho, c_p, C_{CV}\}$ of the two layers are distinguished by subscripts A and B. The coordinate (x, y) is shown in the figure. We number an arbitrary unit-cell as the jth unit-cell. Its left and right boundaries coordinates are $x_L^j=jl$ and $x_R^j=(j+1)l$, respectively; and the coordinate of the interface between layers A and B is $x_{AB}^j=jl+l_A$.

A 1D thermal wave propagates in the periodic structure without any internal heat source or loss (i.e. $Q=0$). For a time-harmonic thermal wave with angular frequency ω, the temperature and heat flux fields may be written as $\{T(x,t),\ q(x,t)\}=\{\hat{T}(x),\ \hat{q}(x)\}e^{-i\omega t}$ with $\hat{T}(x)$ satisfying

$$\hat{T}''(x)+\frac{\omega^2+i\omega/\tau_q}{C_{CV}^2}\hat{T}(x)=0, \tag{4}$$

where $i=\sqrt{-1}$. The general solution of equation (4) is

$$\hat{T}(x)=A_1e^{i\gamma x}+A_2e^{-i\gamma x}, \tag{5}$$

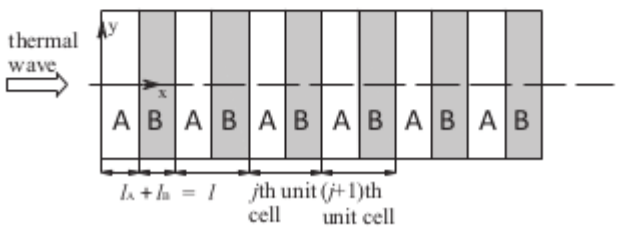


Fig. 1. Schematic diagram of a 1D thermal wave crystal.

where A_1 and A_2 are unknown coefficients, and

$$\gamma=\sqrt{\frac{\omega^2+i\omega/\tau_q}{C_{CV}^2}}, \tag{6}$$

of which the real part demonstrates propagating of the thermal wave and the imaginary part characterizes the attenuation. The heat flux $\hat{q}(x)$ is obtained by following Eq. (2),

$$\hat{q}(x)=-A_1\frac{i\kappa\gamma}{1-i\omega\tau_q}e^{i\gamma x}+A_2\frac{i\kappa\gamma}{1-i\omega\tau_q}e^{-i\gamma x}. \tag{7}$$

For conciseness, the following state vector is introduced,

$$\mathbf{S}(x)=\{\hat{T}(x),\ \hat{q}(x)\}^{\mathrm{T}}=\mathbf{M}(x)\{A_1,\ A_2\}^{\mathrm{T}}, \tag{8}$$

where the superscript T denotes the transpose, and

$$\mathbf{M}(x)=\begin{pmatrix}1 & 1\\ -\frac{i\kappa\gamma}{1-i\omega\tau_q} & \frac{i\kappa\gamma}{1-i\omega\tau_q}\end{pmatrix}\begin{pmatrix}e^{i\gamma x} & 0\\ 0 & e^{-i\gamma x}\end{pmatrix}. \tag{9}$$

$$\mathbf{M}_{\mathrm{Transfer}}^j\mathbf{S}_{\mathrm{AL}}^j=e^{ikl}\mathbf{S}_{\mathrm{AL}}^j, \tag{15}$$

or

$$|\mathbf{M}_{\mathrm{Transfer}}-e^{ikl}\mathbf{I}|=0, \tag{16}$$

where $\mathbf{I}$ is the identity matrix. Considering the detailed expression of each element of the transfer matrix $\mathbf{M}_{\mathrm{Transfer}}$, one can obtain the following concise form of the eigenvalue equation [45]:

$$\cosh(ikl)=\cosh(i\gamma_A l_A)\cosh(i\gamma_B l_B)+\frac{1}{2}\left(\frac{\eta_A\gamma_A}{\eta_B\gamma_B}+\frac{\eta_B\gamma_B}{\eta_A\gamma_A}\right)\sinh(i\gamma_A l_A)\sinh(i\gamma_B l_B), \tag{17}$$

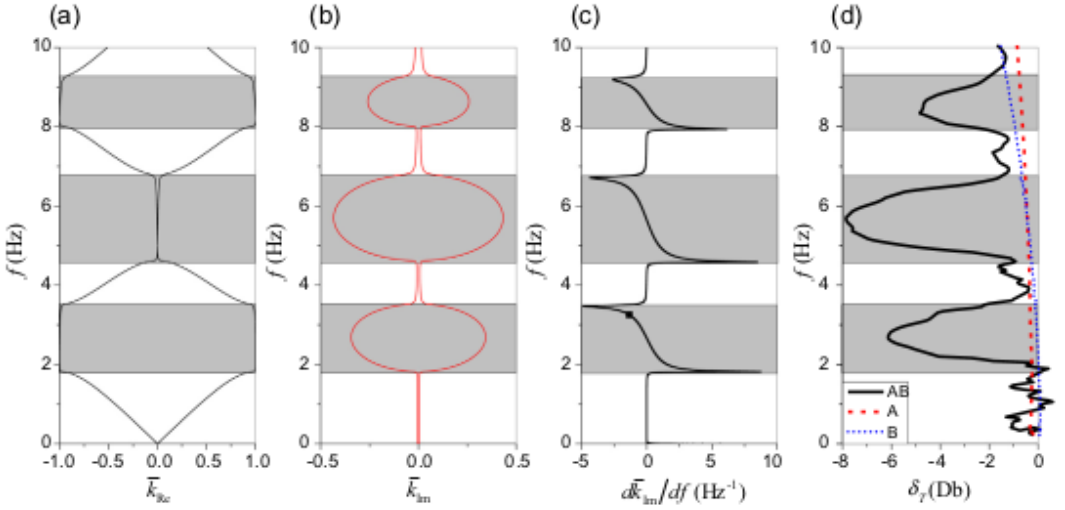


Fig. 2. Complex dispersion curves (with (a) for the real part and (b) for the imaginary part) for the thermal wave propagating in the 1D thermal wave crystal, the normalized wave number is $\bar{k} = k_{Re} + i\bar{k}_{Im} = kl/\pi$; (c) derivative of the imaginary part of the normalized wave number shown in (b), the maxima and minima of the derivative determines the lower and upper edges of the band gaps; (d) the temperature responses δ_T calculated by the FDTD method for the thermal wave crystal of finite thickness (the black solid curve), homogeneous bulk material A (the red dashed curve) and bulk material B (the blue dotted curve), respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

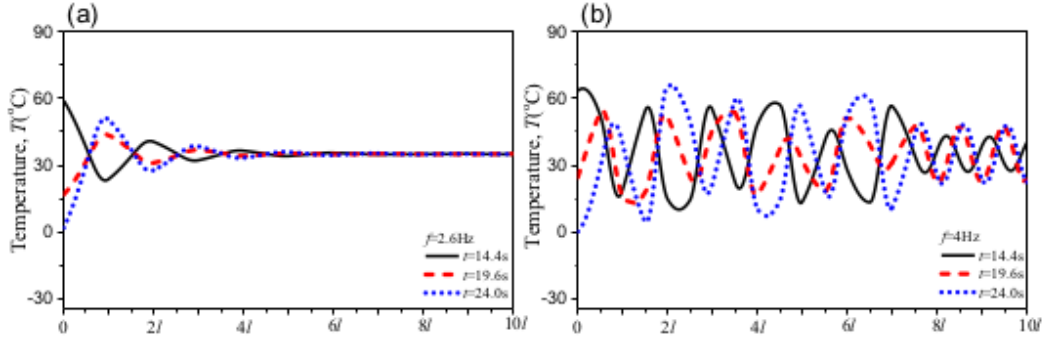


Fig. 3. The temperature distributions at different times in the thermal wave crystal of finite thickness calculated by the FDTD method: (a) $f = 2.6$ Hz within the first band gap, and (b) $f = 4$ Hz outside the band gap.

Complex band structure of thermal wave crystals: The plane-wave method

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In this paper, we present an extension of the plane-wave method (PWM) to compute the complex band structure of thermal wave crystals (TWCs). The structural periodicity of TWC allows the possibility to manipulate non-Fourier heat *via* wave interference. While the Cattaneo-Vernotte (CV) heat conduction theory accurately models oscillatory wave-like propagation of heat in TWCs, obtaining an eigenvalue equation for frequency using the CV wave equation is not possible. To overcome this limitation, we propose a novel approach that solves a complex eigenvalue equation for the Bloch wave vectors.

2. Method

The CV model proposes a modification of Fourier's law of thermal conduction by introducing a time lag between the heat flux vector and the temperature gradient [24,25]. In the case of a one-dimensional periodic structure, the heat conduction model is

$$q(x,t) + \tau(x)\frac{\partial}{\partial t}q(x,t) = -\kappa(x)\frac{\partial}{\partial x}T(x,t), \qquad (1)$$

where $q(x,t)$ and $T(x,t)$ are the time-dependent heat flux and temperature, respectively. There are two position-dependent material parameters, $\tau(x)$ and $\kappa(x)$ that are the relaxation time and thermal conductivity, respectively. The conservation of energy in the absence of heat sources is given by the equation

$$\frac{\partial}{\partial x}q(x,t) = -\rho(x)c_p(x)\frac{\partial}{\partial t}T(x,t), \qquad (2)$$

where $\rho(x)$ and $c_p(x)$ are the position-dependent mass density and specific heat at constant pressure, respectively. By combining Eqs. (1) and (2) we obtain a wave equation in the time-domain as

$$\frac{\partial}{\partial x}\left[\frac{1}{\rho(x)c_p(x)}\frac{\partial}{\partial x}q(x,t)\right] = \frac{1}{\kappa(x)}\frac{\partial}{\partial t}q(x,t) + \frac{\tau(x)}{\kappa(x)}\frac{\partial^2}{\partial t^2}q(x,t). \qquad (3)$$

Considering the Fourier Transform

$$q(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} q(x,\omega)e^{-i\omega t}d\omega, \qquad (4)$$

we obtain a wave equation in the frequency domain

$$\frac{\partial}{\partial x}\left[\frac{1}{\rho(x)c_p(x)}\frac{\partial}{\partial x}q(x,\omega)\right] = -i\omega\frac{1}{\kappa(x)}q(x,\omega) - \omega^2\frac{\tau(x)}{\kappa(x)}q(x,\omega). \qquad (5)$$

The conventional approach of formulating an eigenvalue problem to obtain the frequency as a function of the Bloch wave vector $[\omega(K)]$ is no longer applicable to Eq. (5). In this paper, we propose a method to solve the eigenvalue problem that results from the implementation of the PWM where the Bloch wave vectors $k(\omega)$ are the eigenvalues.

Tunable Photonic Crystals with Semiconducting Constituents

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(Received 24 September 1999)

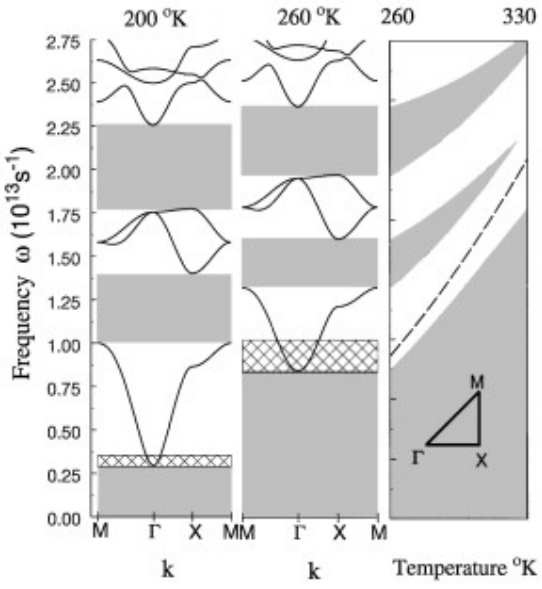
$$\nabla \times \nabla \times \boldsymbol{E} = (\omega/c)^2 \varepsilon(\omega) \boldsymbol{E} = (\omega/c)^2 \varepsilon_0 [1 - \omega_p^2(\boldsymbol{r})/\omega^2] \boldsymbol{E}. \quad (3)$$

By defining $\eta(\boldsymbol{r}) = 1/\varepsilon_0(\boldsymbol{r})$ and $\Omega(\boldsymbol{r}) = \omega_p^2(\boldsymbol{r})/c^2$, Eq. (3) may be written as

$$\eta(\boldsymbol{r}) \nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) + \Omega(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}) = (\omega/c)^2 \boldsymbol{E}(\boldsymbol{r}). \quad (4)$$

$$\sum_{G'} [\eta(\boldsymbol{G} - \boldsymbol{G}') |\boldsymbol{k} + \boldsymbol{G}'|^2 + \Omega(\boldsymbol{G} - \boldsymbol{G}')] E_k(\boldsymbol{G}') = (\omega/c)^2 E_k(\boldsymbol{G}). \quad (5)$$

$$\varepsilon(\omega) = \varepsilon_0 (1 - \omega_p^2/\omega^2).$$



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Temperature tuning of two-dimensional photonic crystals in the presence of phonons and a plasma of electrons and holes

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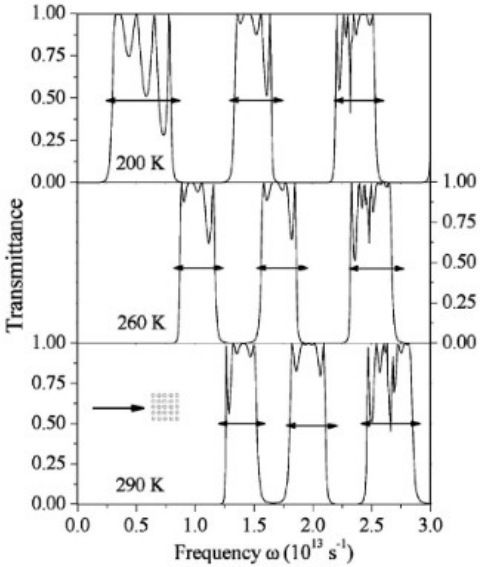
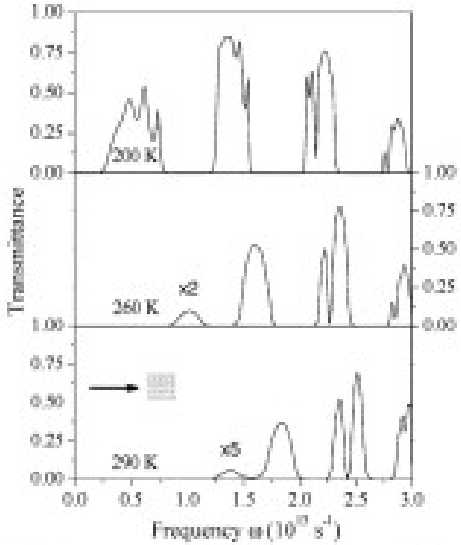
$$\varepsilon(\omega) = \varepsilon_0(1 - \omega_p^2/\omega^2).$$

$$\nabla \times \nabla \times \boldsymbol{E} = (\omega/c)^2 \varepsilon(\omega) \boldsymbol{E}$$

$$\varepsilon(\omega) = \varepsilon_0\left(1 + \frac{\omega_L^2 - \omega_T^2}{\omega_T^2 - \omega^2 - i\omega\gamma} - \frac{\omega_{pe}^2}{\omega(\omega + i/\tau_e)} - \frac{\omega_{ph}^2}{\omega(\omega + i/\tau_h)}\right), \tag{2}$$

$$\frac{\partial}{\partial x}\left[\frac{1}{\rho(x)c_p(x)}\frac{\partial}{\partial x}q(x,\omega)\right] = -i\omega\frac{1}{\kappa(x)}q(x,\omega) - \omega^2\frac{\tau(x)}{\kappa(x)}q(x,\omega).$$





$$\frac{\partial}{\partial x}\left[\frac{1}{\rho(x)c_p(x)}\frac{\partial}{\partial x}q(x,\omega)\right] = -i\omega\frac{1}{\kappa(x)}q(x,\omega) - \omega^2\frac{\tau(x)}{\kappa(x)}q(x,\omega). \quad (5)$$

$$\frac{1}{\rho(x)c_p(x)} = \sum_G \alpha_G e^{iGx},$$

$$\frac{1}{\kappa(x)} = \sum_G \beta_G e^{iGx}$$

$$\frac{\tau(x)}{\kappa(x)} = \sum_G \gamma_G e^{iGx}.$$

$$q(x,\omega) = \sum_G Q_G e^{i(G+K)x},$$

$$\sum_{G'}\left[k^2 A_{G,G'} + kB_{G,G'} + C_{G,G'}\right]Q_{G'} = 0,$$

where the matrix elements are

$$A_{G,G'} = \alpha_{G-G'},$$
$$B_{G,G'} = \alpha_{G-G'}(G+G'),$$
$$C_{G,G'} = GG'\alpha_{G-G'} - i\omega\beta_{G-G'} - \omega^2\gamma_{G-G'}.$$

$$\left(K^2\mathbb{A} + K\mathbb{B} + \mathbb{C}\right)\vec{Q} = 0,$$

$$\begin{pmatrix}\mathbb{C} & \mathbb{B}\\ \mathbb{O} & \mathbb{I}\end{pmatrix}\begin{bmatrix}\vec{Q}\\ K\vec{Q}\end{bmatrix} = K\begin{pmatrix}\mathbb{O} & -\mathbb{A}\\ \mathbb{I} & \mathbb{O}\end{pmatrix}\begin{bmatrix}\vec{Q}\\ K\vec{Q}\end{bmatrix},$$

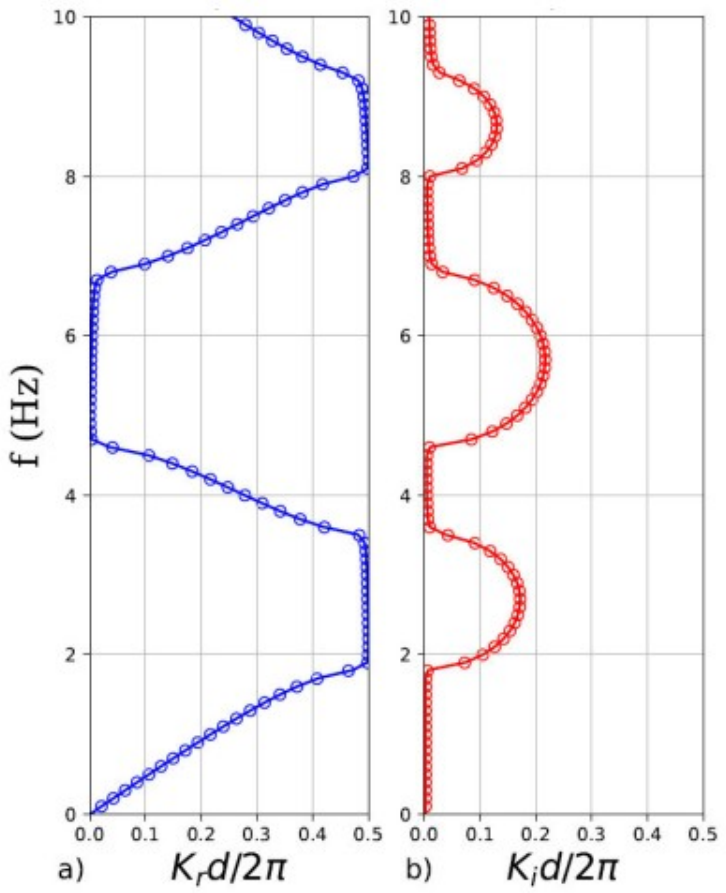


FIGURE 1. Complex band structure of a 1D-TWC, composed of alternating layers of dermis and stratum with equal layer widths and a period of $d = 20$ μm. The solid curves represent the results obtained using the PWM, and the open circles correspond to the TMM calculations. a) The real part of the wave vector is displayed in blue, and b) the imaginary wave vector is displayed in red in panel. We used 101 plane waves for the PWM in this calculation.

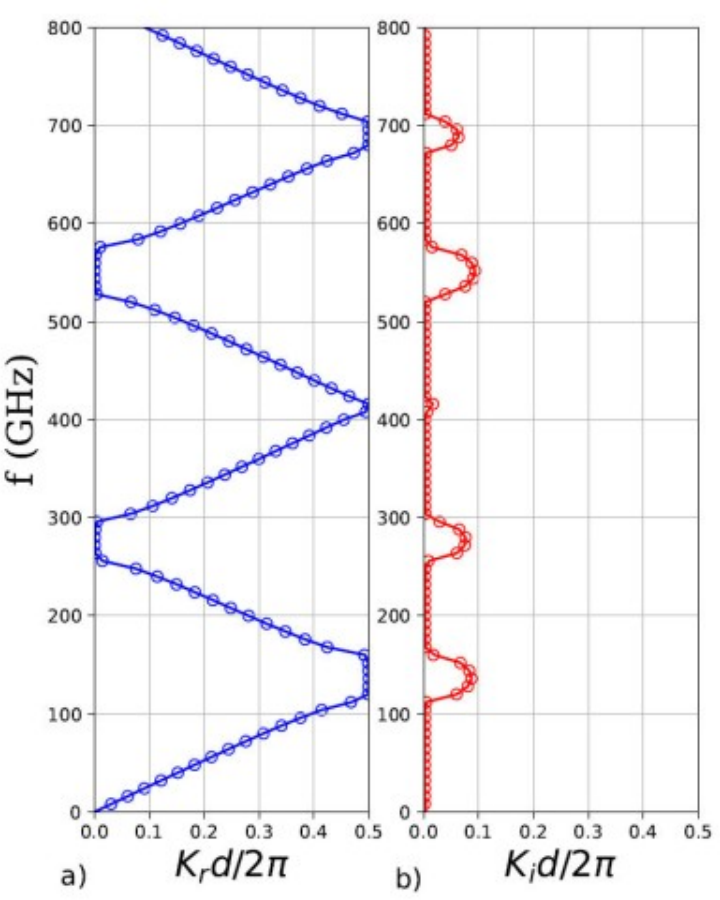


FIGURE 2. Complex band structure of a 1D-TWC, composed of alternating layers of Si and Ge with equal layer widths and a period of $d = 2$ nm. The solid curves represent the results obtained using the PWM, while the open circles correspond to the TMM calculations. a) The real wave vector is displayed in blue, b) while the imaginary wave vector is displayed in red. We used 101 plane waves for the PWM in this calculation.

Complex Band Structure of Two-Dimensional Thermal Wave Crystals

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Complex band structure of two-dimensional thermal wave crystals

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ABSTRACT

We investigate the complex band structure of temperature oscillations in a two-dimensional thermal wave crystal. We use the Cattaneo-Vernotte (CV) heat model to describe the thermal properties. We apply the plane wave method to calculate the complex band structure of a square lattice composed of an infinite array of square bars. We find that a complete band gap exists across the whole Brillouin zone, where temperature oscillations are forbidden. This has potential applications in thermal management, thermal cloaking, and other areas.

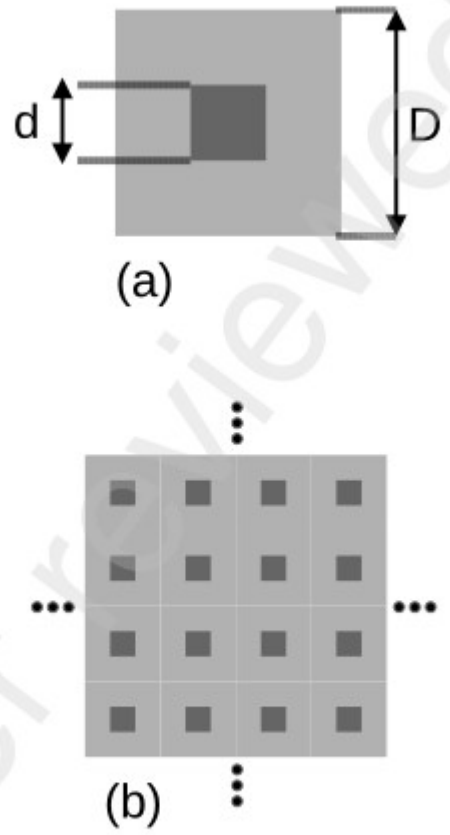

Fig. 1. (a) Square unit cell of side D. The square cylinder at the center has a side d. The dark gray zone at the center corresponds to material a and the surrounding area corresponds to material b. (b) Infinite lattice constructed by the repetition of the unit cell.

2. Theory

The CV heat conduction model proposes a modification of Fourier's law by introducing a time lag into the heat flux vector, in the form [6, 7]

$$\mathbf{q}[\mathbf{x}, t + \tau(\mathbf{x})] = -\kappa(\mathbf{x})\nabla T(\mathbf{x}, t). \tag{1}$$

where $T(\mathbf{x}, t)$, $\mathbf{q}(\mathbf{x}, t)$, $\kappa(\mathbf{x})$ and $\tau(\mathbf{x})$ are the position-dependent temperature, flux vector, thermal conductivity, and time-lag respectively. We expand the left-hand side of Equation (1) using the Taylor series, retaining only the first two terms to have

$$\mathbf{q}(\mathbf{x}, t) + \tau(\mathbf{x})\frac{\partial}{\partial t}\mathbf{q}(\mathbf{x}, t) = -\kappa(\mathbf{x})\nabla T(\mathbf{x}, t). \tag{2}$$

The equation of energy conservation in the absence of internal energy sources is given by

$$\nabla \cdot \mathbf{q}(\mathbf{x}, t) = -\rho(\mathbf{x})c(\mathbf{x})\frac{\partial}{\partial t}T(\mathbf{x}, t), \tag{3}$$

where $\rho(\mathbf{x})$ and $c(\mathbf{x})$ are the position dependent mass den-

$$T(\mathbf{x}, t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} T(\mathbf{x}, \omega)e^{-i\omega t}d\omega$$

$$\mathbf{q}(\mathbf{x}, t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathbf{q}(\mathbf{x}, \omega)e^{-i\omega t}d\omega$$

$$\mathbf{q}(\mathbf{x}, \omega)[1 - i\omega\tau(\mathbf{x})] = -\kappa(\mathbf{x})\nabla T(\mathbf{x}, \omega)$$

$$\nabla \cdot \mathbf{q}(\mathbf{x}, \omega) = i\omega\rho(\mathbf{x})c(\mathbf{x})T(\mathbf{x}, \omega)$$

Combining Eqs. (11) and (12) we obtain

$$\nabla \cdot \alpha(\mathbf{x})\nabla T(\mathbf{x}, \omega) = i\omega\beta(\mathbf{x})T(\mathbf{x}, \omega) \tag{13}$$

where we have defined

$$\alpha(\mathbf{x}) = \frac{\kappa(\mathbf{x})}{i\omega\tau(\mathbf{x}) - 1} \tag{14}$$

and

$$\beta(\mathbf{x}) = \rho(\mathbf{x})c(\mathbf{x}) \tag{15}$$

$$\alpha(\mathbf{x}) = \sum_{\mathbf{G}} \alpha_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} \tag{22}$$

d

$$\beta(\mathbf{x}) = \sum_{\mathbf{G}} \beta_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}} \tag{23}$$

panded in terms of plane waves in the form

$$T(\mathbf{x}, \omega) = \sum_{\mathbf{G}} T_{\mathbf{G}} e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{x}} \tag{30}$$

where $\mathbf{k}$ is a wave vector in the first Brillouin zone. Substitution of Eqs. (22),(23) and (30) in Eq. (13) yields

$$\sum_{\mathbf{G}'} [\alpha_{\mathbf{G}-\mathbf{G}'}(\mathbf{k} + \mathbf{G}) \cdot (\mathbf{k} + \mathbf{G}') + i\omega\beta_{\mathbf{G}-\mathbf{G}'}] T_{\mathbf{G}'} = 0 \tag{31}$$

2.1. $\Gamma \rightarrow \mathbf{X}$ *direction*

In this direction, the wave vector only has a component in the x-direction, $\mathbf{k} = (k_x, 0)$. In this case, it is possible to write the Eq. (31) as

$$\sum_{\mathbf{G}'} \left[k_x^2 A_{\mathbf{G}-\mathbf{G}'} + k_x B_{\mathbf{G}-\mathbf{G}'} + C_{\mathbf{G}-\mathbf{G}'} \right] T_{\mathbf{G}'} = 0 \tag{32}$$

where we have introduced

$$A_{\mathbf{G}-\mathbf{G}'} = \alpha_{\mathbf{G}-\mathbf{G}'}, \tag{33}$$

$$B_{\mathbf{G}-\mathbf{G}'} = \alpha_{\mathbf{G}-\mathbf{G}'}(G_x + G'_x), \tag{34}$$

and

$$C_{\mathbf{G}-\mathbf{G}'} = \alpha_{\mathbf{G}-\mathbf{G}'}\mathbf{G}\cdot\mathbf{G}' + i\omega\beta_{\mathbf{G}-\mathbf{G}'}. \tag{35}$$

The Eq. (32) defines a set of equations that give a matrix equation in the form

$$(k_x^2\mathbf{A} + k_x\mathbf{B} + \mathbf{C})\mathbf{T} = 0 \tag{36}$$

The vector $\mathbf{T}$ has as n coefficients $T_{\mathbf{G}}$ of the Fourier expansion $T(\mathbf{x},\omega)$ defined in Eq. (30). Here $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are matrices with $n \times n$ elements given by $A_{\mathbf{G}-\mathbf{G}'}, B_{\mathbf{G}-\mathbf{G}'}$ and $C_{\mathbf{G}-\mathbf{G}'}$ defined by Eqs. (33-35).

We reformulate the Eq. (32) as a complex eigenvalue problem in the form

$$\begin{pmatrix} \mathbf{C} & \mathbf{B} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{bmatrix} \mathbf{T} \\ k_x\mathbf{T} \end{bmatrix} = k_x \begin{pmatrix} \mathbf{O} & -\mathbf{A} \\ \mathbf{I} & \mathbf{O} \end{pmatrix} \begin{bmatrix} \mathbf{T} \\ k_x\mathbf{T} \end{bmatrix} \tag{37}$$

2.2. $\Gamma \rightarrow \mathbf{M}$ *direction*

In this case, the wave vector is in the $\Gamma \rightarrow \mathbf{M}$ direction. The wave vector is $\mathbf{k} = (k_x, k_y)$, but in this case the k_y component is equal to k_x, $k_y = k_x$. For this direction, we proceed similarly to the previous direction $\Gamma \rightarrow \mathbf{X}$. Starting with Eq. (31) we obtain a set of equations that can be written in the same manner as Eq. (32). The difference is that in this case, the matrix elements are defined by the relations

$$A_{\mathbf{G}-\mathbf{G}'} = \alpha_{\mathbf{G}-\mathbf{G}'} \tag{38}$$

$$B_{\mathbf{G}-\mathbf{G}'} = \frac{1}{2}\alpha_{\mathbf{G}-\mathbf{G}'}(G_x + G'_x + G_y + G'_y) \tag{39}$$

$$C_{\mathbf{G}-\mathbf{G}'} = \frac{1}{2}\alpha_{\mathbf{G}-\mathbf{G}'}\mathbf{G}\cdot\mathbf{G}' + \frac{1}{2}i\omega\beta_{\mathbf{G}-\mathbf{G}'} \tag{40}$$

2.3. $\mathbf{X} \rightarrow \mathbf{M}$ *direction*

In this case, the wave vector is $\mathbf{k} = (0, k_y)$ because is parallel to the y axis. In this case, we start with Eq. (31) and we obtain an eigenvalue equation for k_y in the form

$$(k_y^2\mathbf{A} + k_y\mathbf{B} + \mathbf{C})\mathbf{T} = 0, \tag{41}$$

where the elements of the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$$A_{\mathbf{G}-\mathbf{G}'} = \alpha_{\mathbf{G}-\mathbf{G}'} \tag{42}$$

$$B_{\mathbf{G}-\mathbf{G}'} = \frac{1}{2}\alpha_{\mathbf{G}-\mathbf{G}'}(G_y + G'_y) \tag{43}$$

$$C_{\mathbf{G}-\mathbf{G}'} = \alpha_{\mathbf{G}-\mathbf{G}'}\left[\mathbf{G}\cdot\mathbf{G}' + \frac{\pi}{d}(G_x + G'_x + \left(\frac{\pi}{d}\right)^2\right] + i\omega\beta_{\mathbf{G}-\mathbf{G}'} \tag{44}$$

The Eq. (41) can be written as an eigenvalue problem as the form

$$\begin{pmatrix} \mathbf{C} & \mathbf{B} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{bmatrix} \mathbf{T} \\ k_y\mathbf{T} \end{bmatrix} = k_y \begin{pmatrix} \mathbf{O} & -\mathbf{A} \\ \mathbf{I} & \mathbf{O} \end{pmatrix} \begin{bmatrix} \mathbf{T} \\ k_y\mathbf{T} \end{bmatrix} \tag{45}$$

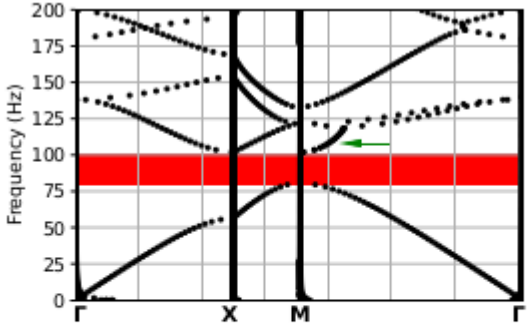

Fig. 2. The complex band structure for thermal waves of a 2D TWC composed of an arrangement of square bars in a square lattice. The bars are composed of materials a and the surrounding background has material b. The filling fraction of material a in the unit cell is $f = 0.7$ We observe a complete band gap with red color.

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Non-Fourier heat conduction in 2D thermal metamaterials

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ticulated as

$$\mathbf{q} + \tau_q \frac{\partial \mathbf{q}}{\partial t} = -\kappa \nabla T \tag{1}$$

where $\mathbf{q}$ symbolizes the heat flux vector, T signifies the temperature, τ_q represents the lagging time between heat flux and temperature, ∇ denotes the gradient, κ and t are the thermal conductivity and the time, respectively. The energy conservation can be expressed by:

$$\nabla \cdot \mathbf{q} = -\rho c_p \frac{\partial T}{\partial t} + Q \tag{2}$$

here Q, ρ, and c_p are the heat source or internal energy generation rate, the mass density, and the specific heat, respectively. Substituting Eq. (2) into Eq. (1), we have the following hyperbolic heat conduction equation without heat source

$$\frac{1}{\tau_q}\frac{\partial T}{\partial t} + \frac{\partial^2 T}{\partial t^2} = C_{cv}^2 \nabla^2 T \tag{3}$$

$$\nabla \cdot \alpha(\mathbf{x}) \nabla T(\mathbf{x}, \omega) = i\omega \beta(\mathbf{x}) T(\mathbf{x}, \omega)$$

where we have defined

$$\alpha(\mathbf{x}) = \frac{\kappa(\mathbf{x})}{i\omega\tau(\mathbf{x}) - 1}$$

and

$$\beta(\mathbf{x}) = \rho(\mathbf{x}) c(\mathbf{x})$$