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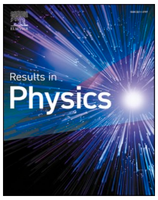

Brewster angle of thermal diffusivity waves at an interface

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ABSTRACT

In this work, we study the existence of the Brewster angle for thermal diffusivity waves. We analyze the propagation of plane thermal waves impinging upon an interface between two media. The condition of zero-reflection defines the Brewster angle. We demonstrate that the Brewster angle only exists for specific combinations of diffusivity and thermal conductivity of the incidence and transmission media.

Introduction

In recent years, the propagation of thermal waves generated by time-dependent heat sources has been analyzed theoretically and experimentally [1–6]. These studies report that the dynamics of a plane thermal wave impinging on an interface follows the traditional characteristics of harmonic analysis at the boundary between two media, such as in the cases of electromagnetic and elastic waves [7,8].

The analysis of thermal waves propagation has been reported over the years. In 1999, M. Bertolotti et al. proved that the reflection and refraction of diffusivity thermal waves follow Snell's law. In 2003, M. L. Shendeleva studied a characterization technique based on the critical angle of plane thermal waves at an interface. [3] In 2017, C. Sanchez-Perez et al. reported experimental evidence of the critical angle for a thermal wave at a solid–liquid interface [5]. Lor and Chu [9] analyzed the influence of thermal resistance on the energy transmission across an interface, finding a threshold value beyond which its effects can be neglected.

The condition for the existence of Brewster's angle at the boundary between two media is that reflection cancels out when the incident wave travels from one medium into the other. For electromagnetic waves, the Brewster angle is a usual topic in textbooks. [10] However, the Brewster angle also exists for other kinds of harmonic waves. In 2000, B. Manzanares-Martinez et al. reported the Brewster angle for mechanical waves in elastic media. [11] For the case of thermal waves, the Brewster angle has not been widely analyzed. To our knowledge, there is only one report in the scientific literature devoted to this subject. In 2011, A. Karam et al. described in a US patent the technological potential for non-destructive characterization via the thermal Brewster angle. [12]

Recently, diffusive thermal wave propagation has attracted considerable attention as a non-destructive technique with applications in thermographic imaging [13], thermal sensing [14], and thermal invisibility [15]. In these physical systems, there is a need to model heat transfer between adjacent media in the direction normal to the interfaces and for all angles of incidence. Angle-dependent analysis of thermal waves is fundamental when system complexities require two-dimensional models to understand the propagating heat flow in complex structures. [16–18].

In this work we analyze the conditions for the existence of the Brewster angle for thermal waves. In this approach we consider an ideal interface, where some effects such as thermal resistance are neglected, and calculate the Brewster angle for several combinations of incident and transmission media.

Theory

Heat conduction is a process described by the diffusion equation

$$\nabla^2 T(\mathrm{x},t) = \frac{1}{D}\frac{\partial}{\partial t}T(\mathrm{x},t) \tag{1}$$

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where $T(\mathrm{x}, t)$ is the time-dependent temperature, and D is the diffusivity of the medium. For harmonic heating with frequency ω, the temperature is periodic in time with a solution in the form $T(\mathrm{x}, t) = \Re\left[\widehat{T}(\mathrm{x})e^{-i\omega t}\right]$ In the frequency domain, the heat diffusion equation is

$$\nabla^2\widehat{T}(\mathrm{x}) = \gamma^2\widehat{T}(\mathrm{x}), \tag{2}$$

where we have introduced the wave number $\gamma = (-1+i)\sqrt{\omega/(2D)}$.

Two basic phenomena observed at the interface between two media are reflection and refraction [19]. In Fig. 1, we consider a thermal plane wave impinging with an incidence angle θ_1 into a plane interface between two media of thermal diffusivities D_1 and D_2 and thermal conductivities κ_1 and κ_2, respectively. The incoming wave generates a reflected and a transmitted wave with angles $\theta_1^{\text{Å}}$ and θ_2, respectively.

The solution of Eq. (2) that describes the heat propagation in a *xy* plane for the incident plane wave is

$$\widehat{T}_1(x, y) = \exp(\gamma_1\cos\theta_1 x + \gamma_1\sin\theta_1 y) \tag{3}$$

The expressions for the reflected $(T_1^{'})$and transmitted (T_2)plane waves can be written as

$$\widehat{T}_1^{'}(x, y) = r\exp\left(\gamma_1\cos\theta_1^{'}x + \gamma_1\sin\theta_1^{'}y\right) \tag{4}$$

and

$$\widehat{T}_2(x, y) = t\exp(\gamma_2\cos\theta_2 x + \gamma_2\sin\theta_2 y) \tag{5}$$

where r and t are the reflection and transmission coefficients, and γ_1 and γ_2 are the wave numbers in medium 1 and 2, respectively. Assuming a perfect contact at the interfase, the boundary conditions are the continuity of temperature and heat flux at $x = 0$:

$$T_1(x, y) + T_1^{'}(x, y) = T_2(x, y) \tag{6}$$

$$\kappa_1\frac{\partial}{\partial x}\left[T_1(x, y) + T_1^{'}(x, y)\right] = \kappa_2\frac{\partial}{\partial x}T_2(x, y) \tag{7}$$

Eqs. (6) and (7) allow us to obtain the thermal Snell's relationships:

$$\theta_1 = \theta_1^{'} \tag{8}$$

$$\frac{1}{\sqrt{D_1}}\sin\theta_1 = \frac{1}{\sqrt{D_2}}\sin\theta_2 \tag{9}$$

Note that in the case of $D_2 > D_1$, Snell's law is valid only below the critical angle defined by $\theta_c = \arcsin\left(\frac{\sqrt{D_2}}{\sqrt{D_1}}\right)$. The reflection coefficient is obtained solving Eqs. (6) and (7),

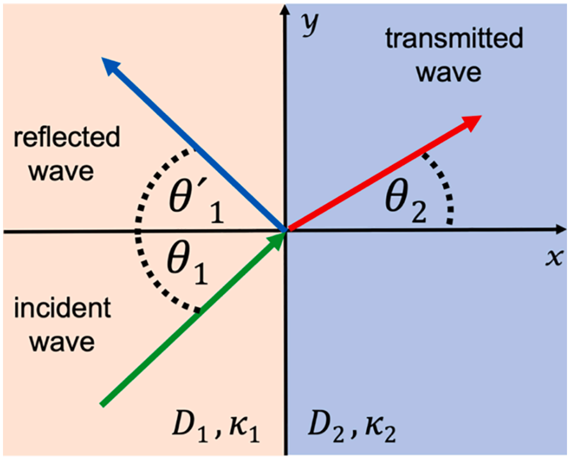

Fig. 1. Reflection and transmission of a plane thermal wave impinging at a plane interface. The angles for the incident, reflected and transmitted thermal waves are θ_1, $\theta_1^{\text{Å}}$ and θ_2, respectively.

$$\frac{(\kappa_1/\sqrt{D_1})cos\theta_1 - (\kappa_2/\sqrt{D_2})cos\theta_2}{r = (\kappa_1/\sqrt{D_1})cos\theta_1 + (\kappa_2/\sqrt{D_2})cos\theta_2} \tag{10}$$

The Brewster angle $(\theta_B)$is defined by the condition $r = 0$. By combining Eqs. (9) and (10) we obtain

$$\cos^2\theta_B = \frac{D_1/D_2 - 1}{\kappa_1^2/\kappa_2^2 - 1} \tag{11}$$

To have a real value for the Brewster angle, $\cos\theta_B$ must be between 0 and 1, and this is only possible under the condition

$$0 < \sqrt{\frac{D_1/D_2 - 1}{\kappa_1^2/\kappa_2^2 - 1}} < 1 \tag{12}$$

The mathematical condition in eq. (12) depends on four variables, D_1, D_2, κ_1 and κ_2. In the next section, we analyze the combination of materials that allows the Brewster angle.

Results

In Table 1, we present the thermal parameters of diffusivity and thermal conductivity of ten materials: Nickel, Chromium, Germanium, Zinc, Tungsten, Silicon, Aluminum, Copper, Gold and Silver. In Table 2, we analyze the existence of the Brewster angle for all interface combinations, where the first column contains the incident medium, and the first row contains the transmission medium. Thus, each cell corresponds to a specific interface of incidence and transmission media, and the cell value is the Brewster angle obtained with equation (11). When a particular combination of media has no Brewster angle, the cell is colored red, whereas a gray cell means that the transmission and incidence media combination define a continuous medium. We note that Brewster's angle occurs more frequently in some materials than in others. For example, with Aluminum as the incident medium, we have a Brewster angle for all possible transmission media except, of course, itself. In contrast, for Silicon, we have four interfaces where the Brewster angle does not exist.

To understand the existence (or not) of the Brewster angle, we present in Fig. 2 the case where the incident medium is Si. Panel (a) shows the reflection when the transmission medium is Ge, Al, Cu, Au, or Ag, using blue, red, orange, cyan, and magenta lines, respectively. The Brewster angle corresponds to the point where the reflection cancels out, represented by a filled circle on the line $r = 0$. We also note that for these metals, the reflection becomes 1 at the critical angle because the diffusivity of the transmission medium is higher than the incident medium (Si). Panel (b) shows no Brewster angle for materials such as Ni, Cr, Zn, and W, represented with blue, green, red, and cyan colors, respectively.

Conclusions

In conclusion, we have found the conditions to have a Brewster angle for thermal waves. We found that only interfaces with a specific

Table 1
Material thermal parameters.

Medium	D (10^{-6} m^2/s)	κ (W/mK)
Nickel	22.95	90.7
Chromium	29.15	93.7
Germanium	34.71	59.9
Zinc	41.76	116
Tungsten	68.30	174
Silicon	89.21	148
Aluminum	93.28	238
Copper	116.60	401
Gold	127.32	317
Silver	173.86	429

Table 2
Brewster angle (in degrees) for different combinations of incidence and transmission media. The first column represents the incident medium, while the first row represents the transmitted medium. Cells in red correspond to pairings where there is no Brewster angle.

		Transmission medium									
		Ni	Cr	Ge	Zn	W	Si	Al	Cu	Au	Ag
Incidence medium	Ni		🟥	🟥	🟥	17.28	🟥	20.08	23.06	19.11	17.59
	Cr	🟥		🟥	21.22	26.03	🟥	25.57	27.04	23.19	20.78
	Ge	🟥	🟥		61.32	41.67	31.26	35.04	32.05	29.71	25.38
	Zn	🟥	25.68	74.23		33.24	🟥	31.66	33.18	28.25	25.12
	W	30.83	42.21	68.85	44.51		🟥	40.67	44.40	35.46	31.51
	Si	🟥	🟥	56.31	🟥	🟥		74.53	58.56	51.78	41.97
	Al	43.81	50.56	70.25	51.68	49.60	80.23		56.24	38.48	35.08
	Cu	62.02	65.40	76.58	66.13	66.10	77.28	68.26		🟥	🟥
	Au	50.44	55.40	71.68	55.74	52.38	69.81	46.63	🟥		39.83
	Ag	56.31	60.08	73.60	60.03	56.25	69.02	51.70	🟥	48.46	

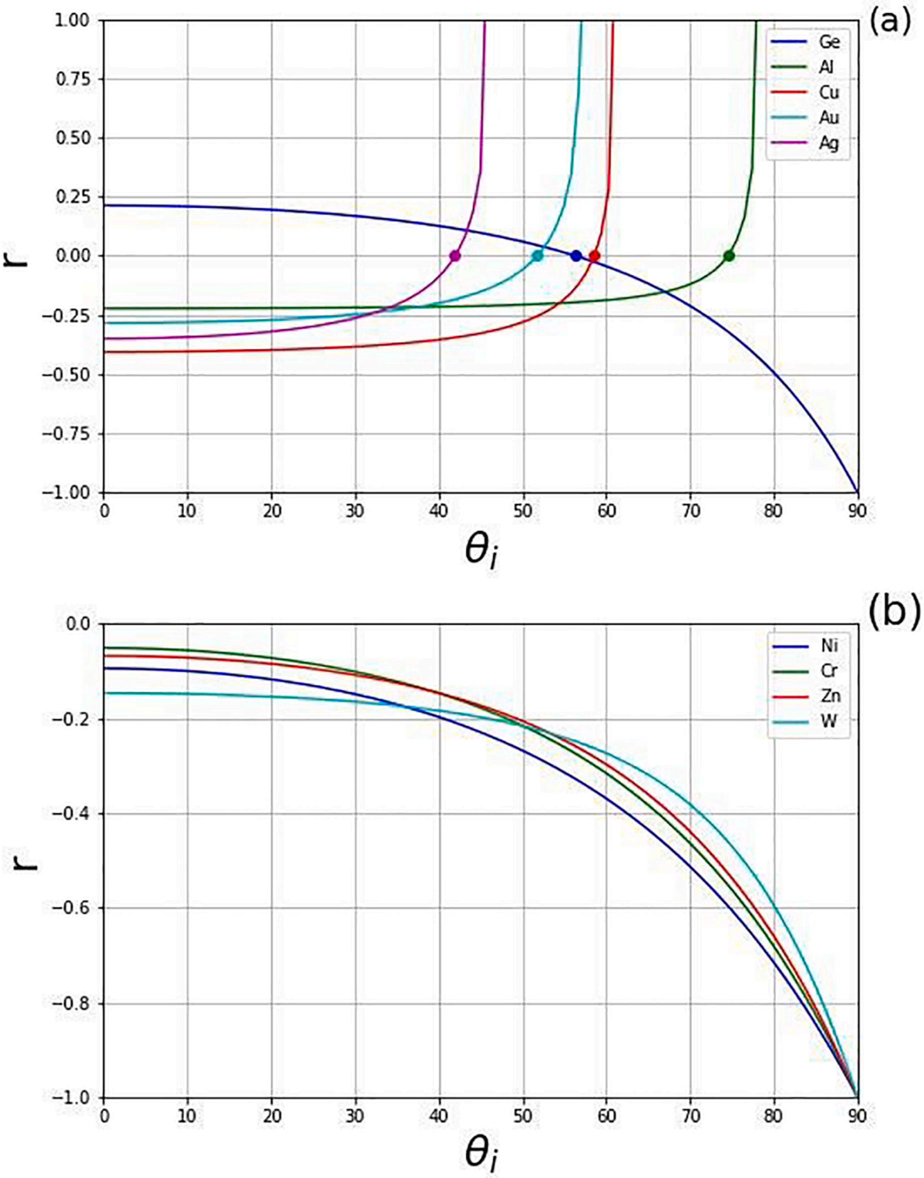

Fig. 2. Reflection coefficient for a thermal wave impinging from silicon into different media. Panel (a) shows the cases where Brewster angle exists, represented by a filled circle on the line r = 0. In panel (b), it can be observed that there is no Brewster angle.

combination of thermal diffusivity and conductivity present the Brewster angle. Similarly, due to their thermal parameters, some interfaces do not have a Brewster angle. We hope that our analysis will be helpful to explain the existence or not of the thermal Brewster angle.

CRediT authorship contribution statement

Cesar Augusto Romero-Ramos: Formal analysis, Investigation. **Jesús Manzanares-Martínez:** Conceptualization, Formal analysis. **Betsabe Manzanares-Martínez:** Formal analysis, Validation. **Diego Soto-Puebla:** Writing - review & editing, Formal analysis. **Gerardo Alejandro Morales-Morales:** Writing - original draft, Software, Methodology. **Carlos Eduardo Ruiz-Rosales:** Validation, Data curation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Tzou DY. Reflection and refraction of thermal waves from a surface or an interface between dissimilar materials. Int J Heat Mass Transf 1993;36(2):401–10.

[2] Bertolotti M, Liakhou G, Li Voti R, Paoloni S, Sibilia C. Thermal wave reflection and refraction: Theoretical and experimental evidence. J Appl Phys 1999;85(7):3540–5.

[3] Shendeleva ML. Determination of thermal diffusivity using the critical angle in thermal wave reflection at an interface. Rev Sci Instrum 2003;74(1):430–2.

[4] Shendeleva ML. Thermal wave reflection and refraction at a plane interface: Two-dimensional geometry. Physical Review B 2002;65(13):134209.

[5] Sánchez-Pérez C, De León-Hernández A, García-Cadena C. Optical device for thermal diffusivity determination in liquids by reflection of a thermal wave. Rev Sci Instrum 2017;88(8):084901.

[6] Nie BD, Cao BY. Reflection and refraction of a thermal wave at an ideal interface. Int J Heat Mass Transf 2018;116:314–28.

[7] Jackson JD. Classical electrodynamics. Wiley; 1999.

[8] L D Landau L. P. Pitaevskii A. M. Kosevich E.M. Lifshitz.. theory of elasticity. 3rd edition. Butterworth-Heinemann; 1984.

[9] Lor W, Chu H. Effect of interface thermal resistance on heat transfer in a composite medium using the thermal wave model. Int J Heat Mass Transf 2000;43:653–63.

[10] Born M, Wolf E. Principles of optics: electromagnetic theory of propagation, interference and diffraction of light. Elsevier; 2013.

[11] Manzanares-Martínez B, Ramos-Mendieta F. Transverse elastic waves in superlattices: The brewster acoustic angle. Physical Review B 2000;61(19):12877.

[12] Karam, M.A., Volk, C.H., Meyer, A.D. Systems and methods for measuring at least one thermal property of materials based on a thermal brewster angle. 2011. US Patent 8,065,108.

[13] Gavrilov D, Maev RG, Almond D. A review of imaging methods in analysis of works of art: Thermographic imaging method in art analysis. Can J Phys 2014;92(4):341–64.

[14] Reyes-Romero D, Behrmann O, Dame G, Urban G. Dynamic thermal sensor for biofilm monitoring. Sens Actuators, A 2014;213:43–51.

[15] Farhat M, Chen PY, Bagci H, Amra C, Guenneau S, Alu A. Thermal invisibility based on scattering cancellation and mantle cloaking. Sci Rep 2015;5(1):1–9.

[16] Qiu L, Guo P, Kong Q, Tan CW, Liang K, Wei J, et al. Coating-boosted interfacial thermal transport for carbon nanotube array nano-thermal interface materials. URL Carbon 2019;145:725–33. https://www.sciencedirect.com/science/article/pii/S0008622319300880.

[17] Yu W, Liu C, Qiu L, Zhang P, Ma W, Yue Y, et al. Advanced thermal interface materials for thermal management. Engineered Science 2018;2(9):1–3.

[18] Qiu L, Scheider K, Radwan SA, Larkin LS, Saltonstall CB, Feng Y, et al. Thermal transport barrier in carbon nanotube array nano-thermal interface materials. Carbon 2017;120:128–36.

[19] Almond DP, Patel P, Patel P. Photothermal science and techniques, vol. 10. Springer Science & Business Media; 1996.