LXVIII Congreso Nacional de Física

12 al 17 de octubre del 2025 en la Ciudad de Toluca, Estado de México, en el Centro de Convenciones

Complex band structure of two-dimensional thermal wave crystals

We investigate the complex band structure of temperature oscillations in a two-dimensional thermal wave crystal. We use the CattaneoVernotte heat model to describe the thermal properties. We apply the plane wave method to calculate the complex band structure of a square lattice composed of an infinite array of square bars. We find that a complete band gap exists across the first Brillouin zone, where temperature oscillations are forbidden. This has potential applications in thermal management, thermal cloaking, and other areas.

[1] C. A. Romero-Ramos, J. Manzanares-Martinez, D. Soto-Puebla, and B. Manzanares Martínez, "Complex band structure of two-dimensional thermal wave crystals", Rev. Mex. Fís., vol. 70, no. 6 Nov-Dec, pp. 061603 1–, Nov. 2024.
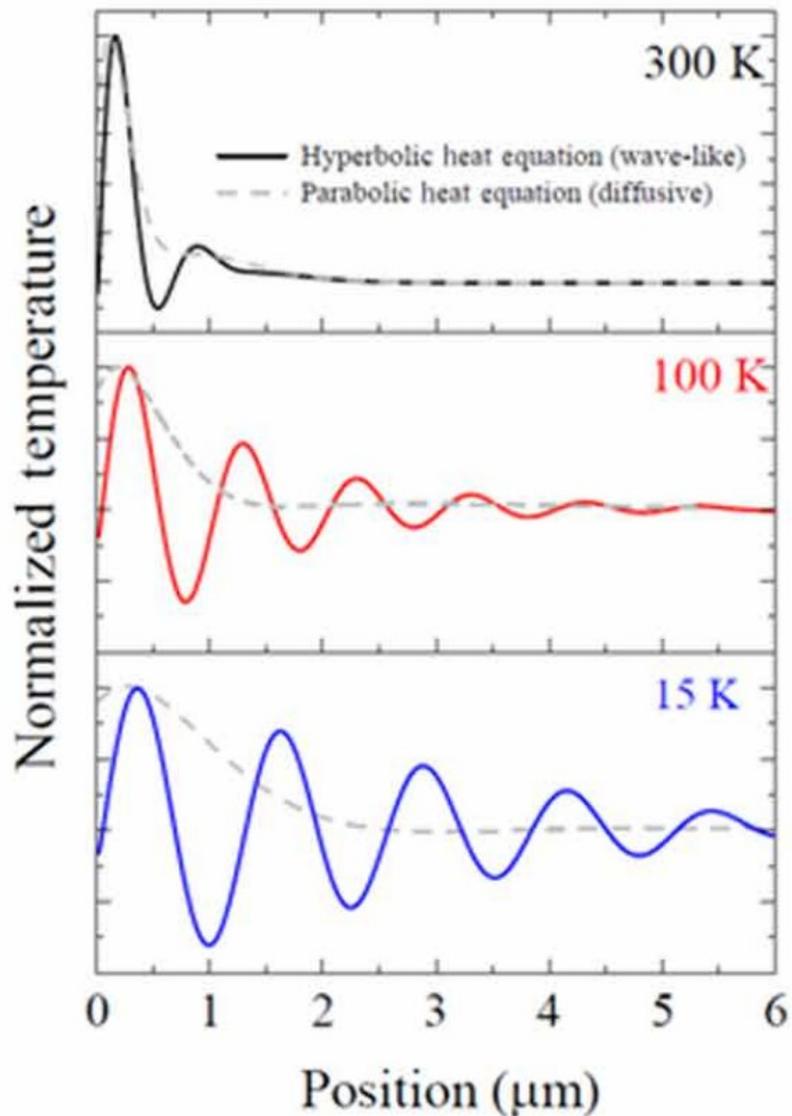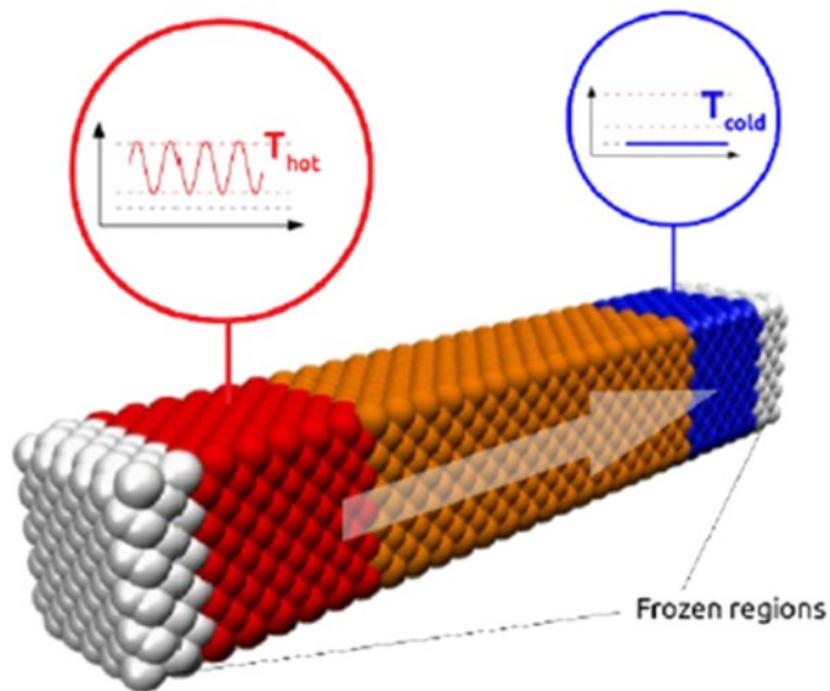
Jesus Manzanares Martinez

Universidad de Sonora

Que son las ondas termicas?

Observation of second sound in a rapidly varying temperature field in Ge

El modelo de Cattaneo-Vernotte

The Cattaneo-Vernotte (CV) heat-conduction model proposes a modification to the Fourier law as follows: [25,26]

$$q(x,t) + \tau \frac{\partial}{\partial t} q(x,t) = -\kappa \frac{\partial}{\partial x} T(x,t) \tag{1}$$

where q is the heat flux, κ is the thermal conductivity, τ is a time-lag, and T is the temperature. The energy conservation equation in absence of heat sources is given by [25,26]

$$\frac{\partial}{\partial x} q(x,t) = -\rho c_p \frac{\partial}{\partial t} T(x,t), \tag{2}$$

where ρ is the mass density and c_p is the specific heat at constant pressure. We combine Eqs. (1) and (2) to obtain [8]

$$\tau \frac{\partial^2}{\partial t^2} T(x,t) + \frac{\partial}{\partial t} T(x,t) = D \frac{\partial^2}{\partial x^2} T(x,t). \tag{3}$$

where $D = \kappa/(\rho c_p)$ is the thermal diffusivity. This diferential equation describes wave-like heat transport according to the CV model [25,26]. The solutions are propagating waves of temperature with a damping term given by the term $\partial T/\partial t$.

(G. Morales-Morales and J. Manzanares-Martinez)

Solution of the wave equation in the frequency domain

For a harmonic thermal wave, the Fourier transform allows switching from the time domain into the frequency domain. The Fourier transform for the temperature and heat flux are [27]

$$T(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} T(x,\omega) e^{-i\omega t} d\omega \qquad (4)$$

and

$$q(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} q(x,\omega) e^{-i\omega t} d\omega. \qquad (5)$$

In the frequency domain, Eqs. (1) and (2) are

$$q(x,\omega) = \frac{-\kappa}{(1-i\omega\tau)} \frac{\partial}{\partial x} T(x,\omega) \qquad (6)$$

and

$$T(x,\omega) = \frac{1}{i\omega\rho c_p} \frac{\partial}{\partial x} q(x,\omega). \qquad (7)$$

Combining Eqs. (6) and (7) we obtain a Helmholtz equation in the form

$$\frac{\partial}{\partial x^2} T(x,\omega) = -\gamma^2 T(x,\omega), \qquad (8)$$

where we have introduced a wave vector in the form

$$\gamma^2 = \frac{\omega^2}{v^2} \left(1 + \frac{i}{\omega\tau}\right). \qquad (9)$$

The parameter v is the speed of propagation of the thermal wave

$$v = \sqrt{\frac{\kappa}{\rho\tau c_p}}. \qquad (10)$$

The general solution of Eq. (8) is

$$T(x,\omega) = t^+ e^{i\gamma x} + t^- e^{-i\gamma x}. \qquad (11)$$

On the right side we have two solutions, the first and second terms are waves propagating to the right and left, respectively [27]. The heat flux can be obtained by combining Eqs. (6) and (11) to obtain

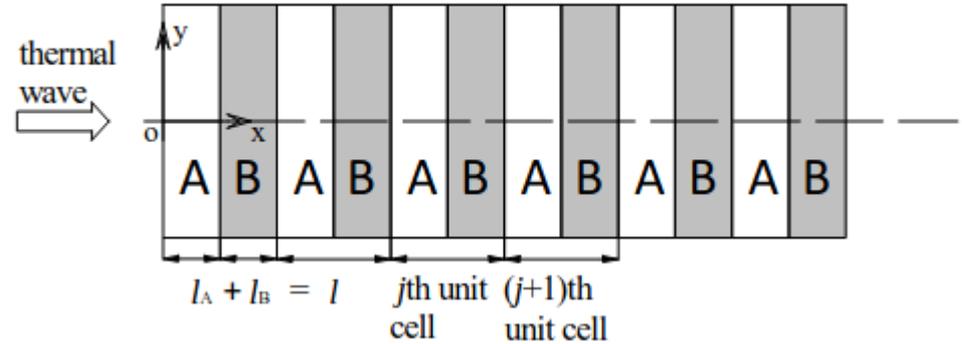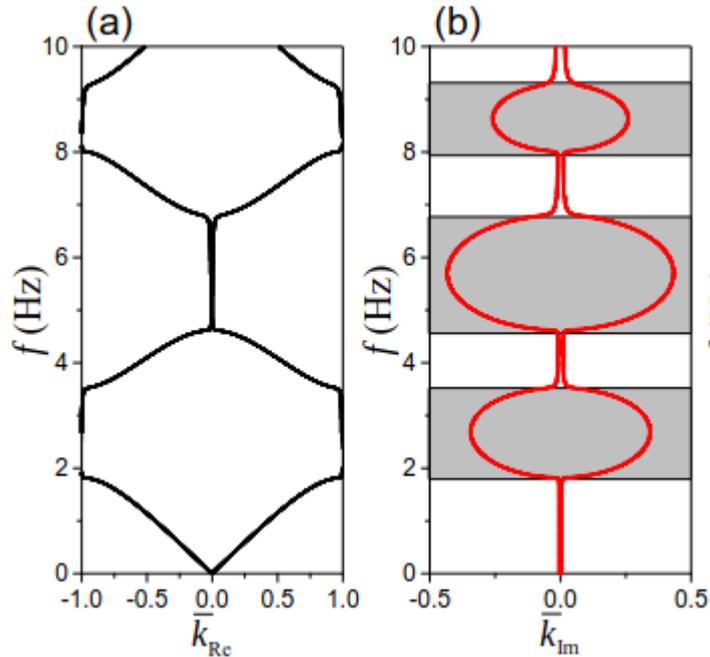$$q(x,\omega) = Z \left(t^+ e^{i\gamma x} - t^- e^{-i\gamma x}\right), \qquad (12)$$

where we have introduced the thermal impedance

$$Z = \frac{-i\gamma\kappa}{1 - i\omega\tau}. \qquad (13)$$

(G. Morales-Morales and J. Manzanares-Martinez)

Heat reduction by thermal wave crystals

A-Li Chen [a] ✉, Zheng-Yang Li [a b], Tian-Xue Ma [a b], Xiao-Shuang Li [a], Yue-Sheng Wang [a]

✉

TABLE 1 Material constants [42]

Component materials	Stratum-like (Layer A)	Dermis-like (Layer B)
Thermal conductivity (W/(m·K))	$\kappa_A = 0.235$	$\kappa_B = 0.445$
Specific heat (J/(kg·K))	$c_{pA} = 3600$	$c_{pB} = 3300$
Density (kg/m³)	$\rho_A = 1500$	$\rho_B = 1116$
Relaxation time (s)	$\tau_{qA} = 1$	$\tau_{qB} = 20$

$$\cosh(i\pi\bar{k}) = \cosh(i n_A \bar{\gamma}_A \bar{\omega})\cosh(i n_B \bar{\gamma}_B \bar{\omega}\sqrt{\bar{c}_{\rho n}/\bar{\kappa}_n})$$

$$+\frac{1}{2}\left(\frac{1}{\sqrt{\bar{c}_{\rho n}\bar{\kappa}_n}}\frac{\bar{\gamma}_B}{\bar{\gamma}_A} + \sqrt{\bar{c}_{\rho n}\bar{\kappa}_n}\frac{\bar{\gamma}_A}{\bar{\gamma}_B}\right)\sinh(i n_A \bar{\gamma}_A \bar{\omega})\sinh(i n_B \bar{\gamma}_B \bar{\omega}\sqrt{\bar{c}_{\rho n}/\bar{\kappa}_n}),$$

Heat reduction by thermal wave crystals

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Complex band structure of thermal wave crystals: The plane-wave method

C. A. Romero-Ramos[a], M. B. Manzanares-Martinez[b], D. Soto-Puebla[c] and J. Manzanares-Martinez[c]

$$\frac{\partial}{\partial x}\left[\frac{1}{\rho(x)c_p(x)}\frac{\partial}{\partial x}q(x,\omega)\right]$$

$$= -i\omega\frac{1}{\kappa(x)}q(x,\omega) - \omega^2\frac{\tau(x)}{\kappa(x)}q(x,\omega).$$

$$\frac{1}{\rho(x)c_p(x)} = \sum_G \alpha_G e^{iGx},$$

$$\frac{1}{\kappa(x)} = \sum_G \beta_G e^{iGx}$$

$$\frac{\tau(x)}{\kappa(x)} = \sum_G \gamma_G e^{iGx}.$$

$$q(x,\omega) = \sum_G Q_G e^{i(G+K)x},$$

$$\sum_{G'}\left[k^2 A_{G,G'} + k B_{G,G'} + C_{G,G'}\right]Q_{G'} = 0, \qquad (10)$$

where the matrix elements are

$$A_{G,G'} = \alpha_{G-G'},$$

$$B_{G,G'} = \alpha_{G-G'}(G+G'),$$

$$C_{G,G'} = GG'\alpha_{G-G'} - i\omega\beta_{G-G'} - \omega^2\gamma_{G-G'}. \qquad (11)$$

If all the points of the reciprocal lattice are considered, then we obtain an infinite set of equations that is convenient to write in the form

$$\left(K^2\mathbb{A} + K\mathbb{B} + \mathbb{C}\right)\vec{Q} = 0, \qquad (12)$$

$$\begin{pmatrix}\mathbb{C} & \mathbb{B} \\ \mathbb{O} & \mathbb{I}\end{pmatrix}\begin{bmatrix}\vec{Q} \\ K\vec{Q}\end{bmatrix} = K\begin{pmatrix}\mathbb{O} & -\mathbb{A} \\ \mathbb{I} & \mathbb{O}\end{pmatrix}\begin{bmatrix}\vec{Q} \\ K\vec{Q}\end{bmatrix},$$

a) $K_r d/2\pi$

b) $K_i d/2\pi$

(a) $\bar{k}_{Rc}$

(b) $\bar{k}_{Im}$
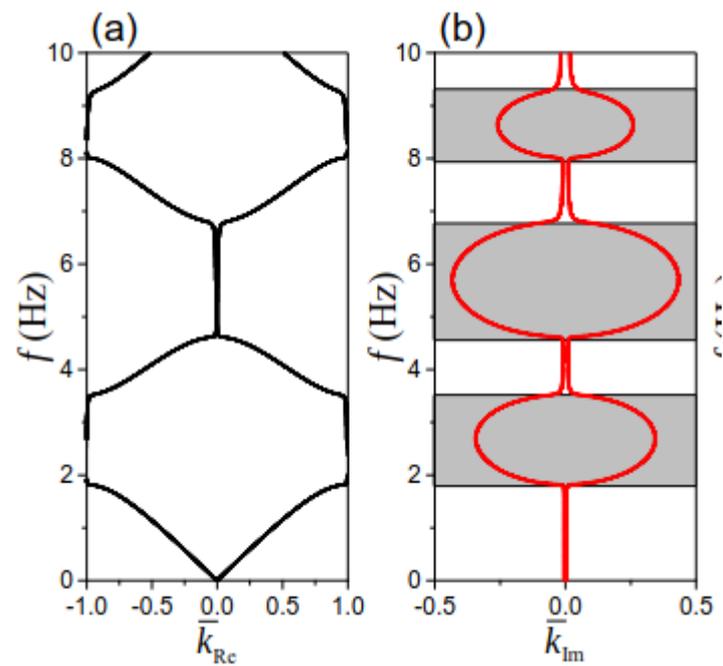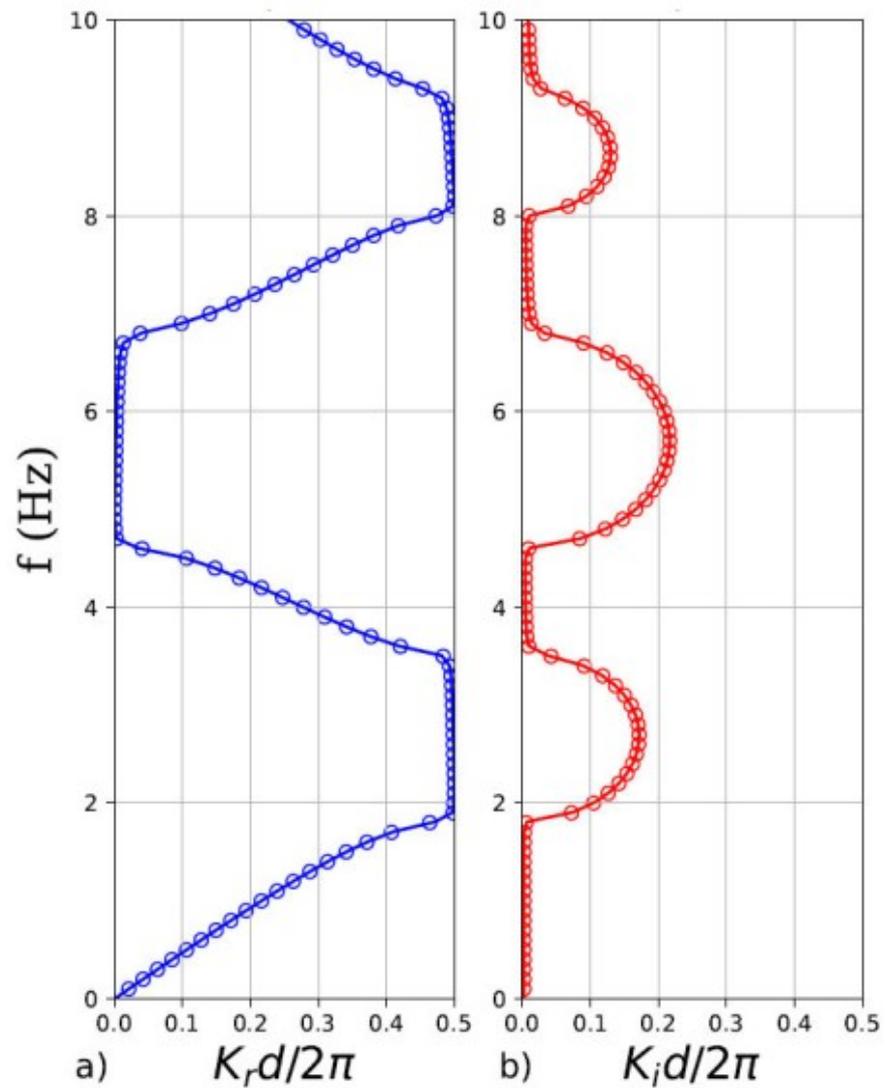
Temperature tuning of two-dimensional photonic crystals in the presence of phonons and a plasma of electrons and holes

J. Manzanares-Martínez and F. Ramos-Mendieta

Centro de Investigación en Física de la Universidad de Sonora, Apartado Postal 5-088, Hermosillo, Sonora 83190, Mexico
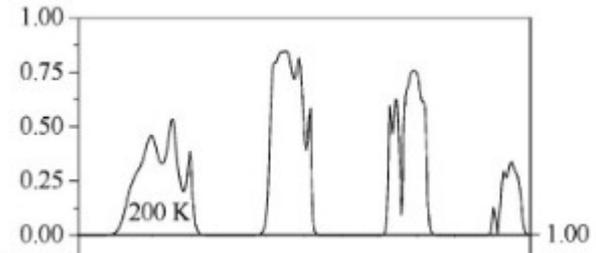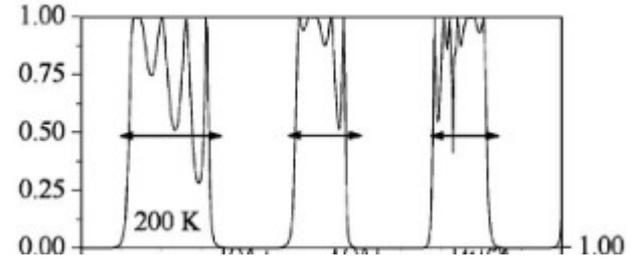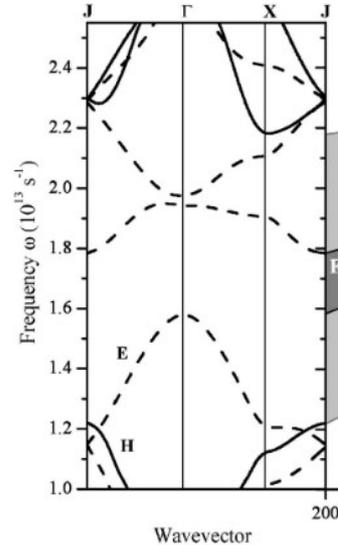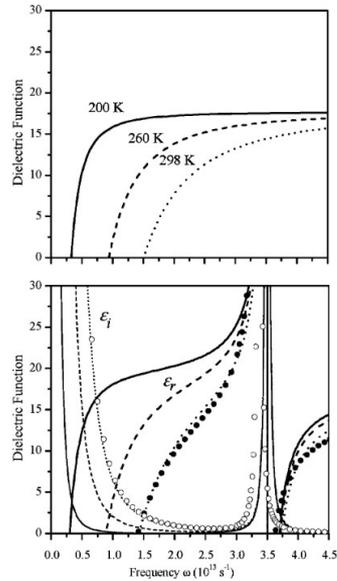
P. Halevi

Instituto Nacional de Astrofísica, Óptica y Electrónica, Apartado Postal 51, Puebla, Puebla 72000, Mexico

$$\varepsilon(\omega) = \varepsilon_0 \left(1 + \frac{\omega_L^2 - \omega_T^2}{\omega_T^2 - \omega^2 - i\omega\gamma} - \frac{\omega_{pe}^2}{\omega(\omega + i/\tau_e)} - \frac{\omega_{ph}^2}{\omega(\omega + i/\tau_h)} \right),$$

$$\sum_{G'} \{\Omega(G - G') + \eta(G - G')|k + G'|^2\} E_z(G') = \left(\frac{\omega}{c}\right)^2 E_z(G).$$

Complex band structure of two-dimensional thermal wave crystals

C. A. Romero-Ramos, J. Manzanares-Martinez*, and D. Soto-Puebla
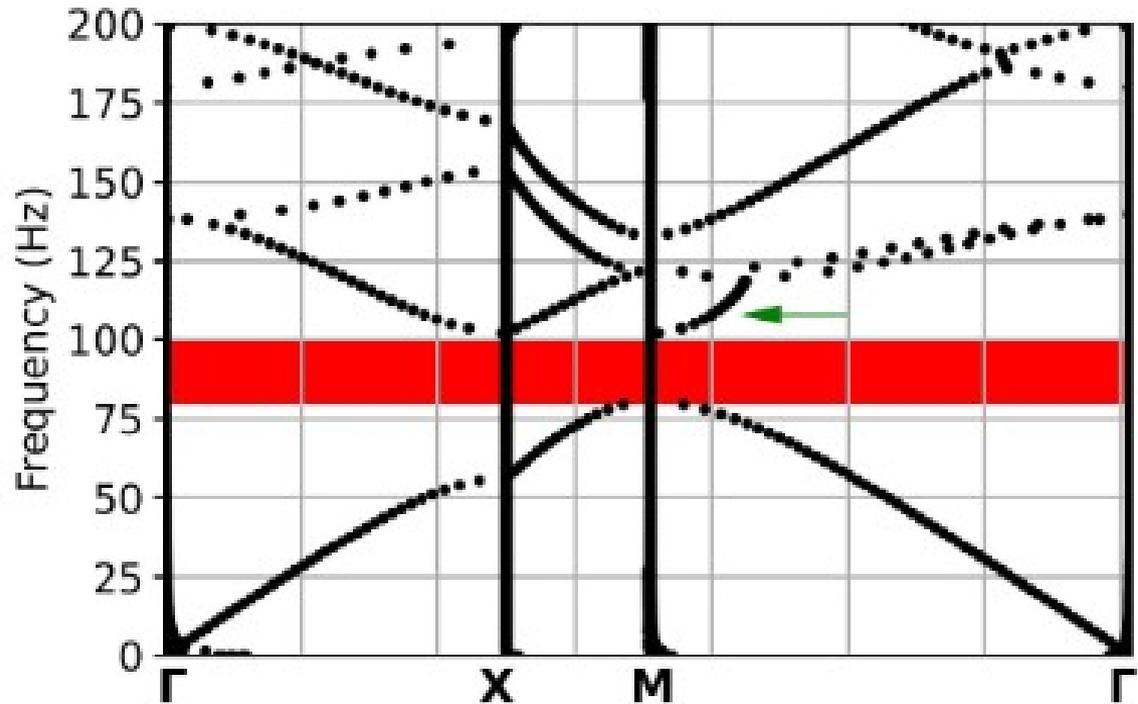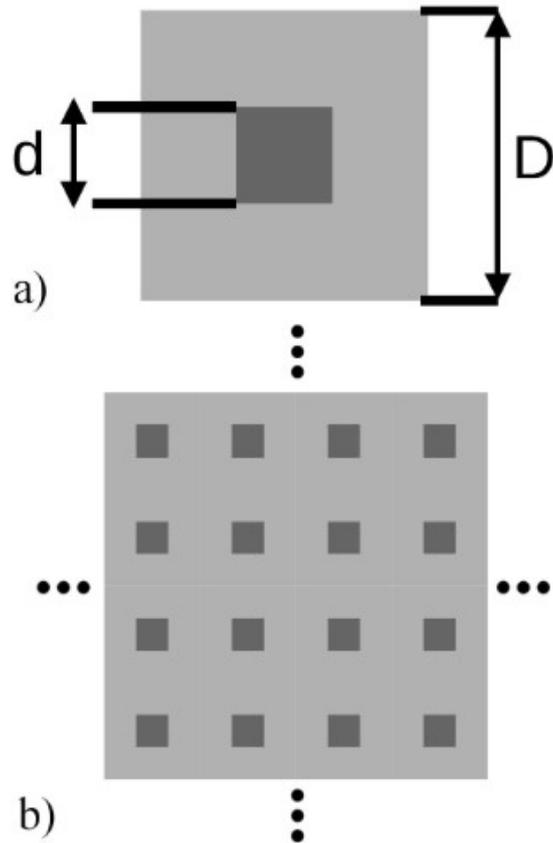
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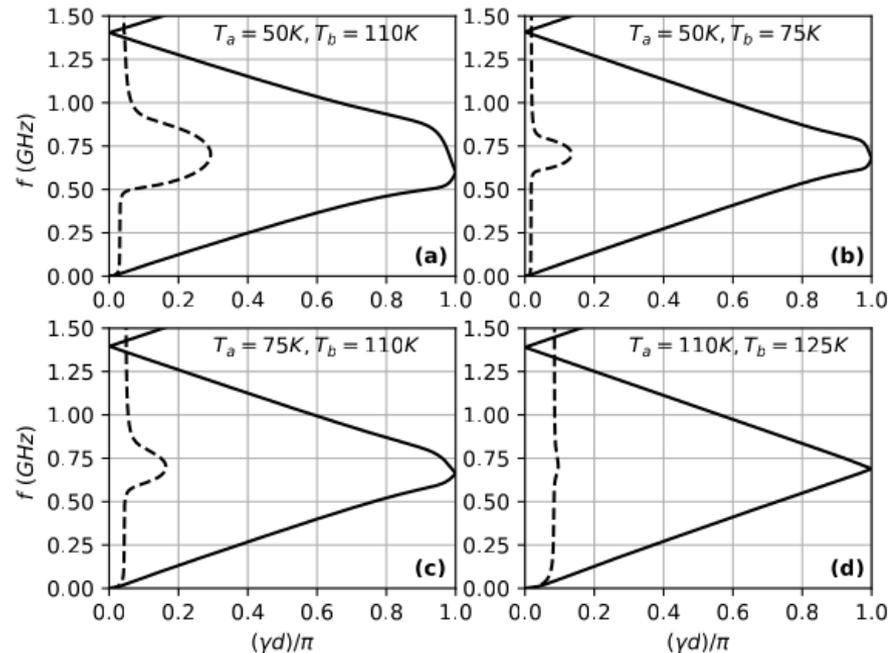
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